Model-Based Estimation of Active Knee Stiffness

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Abstract—Knee joint impedance varies substantially during physiological gait. Quantifying this modulation is critical for the design of transfemoral prostheses that aim to mimic physiological limb behavior. Conventional methods for quantifying joint impedance typically involve perturbing the joint in a controlled manner, and describing impedance as the dynamic relationship between applied perturbations and corresponding joint torques. These experimental techniques, however, are difficult to apply during locomotion without impeding natural movements. In this paper, we propose a method to estimate the elastic component of knee joint impedance that depends on muscle activation, often referred to as active knee stiffness. The method estimates stiffness using a musculoskeletal model of the leg and a model for activation-dependent short-range muscle stiffness. Muscle forces are estimated from measurements including limb kinematics, kinetics and muscle electromyograms. For isometric validation, we compare model estimates to measurements involving joint perturbations; measured stiffness is 17 % lower than model estimates for extension, and 42 % lower for flexion torques. We show that sensitivity of stiffness estimates to common approaches for estimating muscle force is small in isometric conditions. We also make initial estimates of how knee stiffness is modulated during gait, illustrating how this approach may be used to obtain parameters relevant to the design of transfemoral prostheses.

I. INTRODUCTION

The mechanical properties of the human leg can be tuned to maximize performance over a wide range of behaviors, and even during different phases of a single behavior. For example, during locomotion, the leg must be sufficiently stiff to prevent buckling at heel strike, yet also sufficiently compliant to allow for an effortless swing phase. Understanding how leg mechanics are modulated across different tasks, including locomotion, is an important problem in human motor control. It is also critical for understanding how to design artificial legs that can begin to replicate the capabilities of the unimpaired human lower limb. Current approaches for regulating knee stiffness in artificial devices are largely heuristic [1], since quantitative estimates of how knee stiffness is modulated during locomotion are not available. Here we propose a method for obtaining those estimates in a manner that does not impede natural locomotor behaviors.

The mechanical properties of a limb and the joints within that limb can be characterized by their mechanical impedance.

The impedance of a joint describes the dynamic relationship between an externally induced movement of a joint and the torques required to effect that movement [2]. For small displacements, impedance can be described by the inertial, viscous and elastic properties of the joint [3]. The viscous and elastic properties of a joint arise from the passive properties of the anatomical structures within and surrounding the joint, as well as from the activation-dependent properties of the muscles spanning the joint [4], [5]. Joint viscoelasticity can be changed through changes in muscle activation, as determined from previous experiments [6], [7]. The activation-dependent elasticity, or stiffness, of the muscles spanning the joint is thought to be closely linked to the short-range stiffness properties of muscle [8]. In many cases, there appears to be a close relationship between muscle or joint stiffness and the corresponding viscosity [3], suggesting that if one can be estimated, the other can be at least approximately inferred.

Experiments to determine joint stiffness typically involve perturbing the joint in a controlled manner and measuring the corresponding motions and torques. These methods have been applied to assess passive knee joint stiffness (i.e. when muscles are not activated) [2], [9], [10]; research on active joint stiffness (i.e. joint stiffness attributed to activation-dependent muscle elasticity) for the lower limb mainly focused on the ankle joint, for example investigating dependence of stiffness on ankle angle [11], ankle torque [6], [12], influence of co-contraction [13] or stiffness variation due to non-isometric contractions [14]. Fewer studies have been published on active knee joint stiffness. Those that have been completed considered only isometric conditions, and reported similar findings to those observed at the ankle [7], [15].

Direct estimates of knee stiffness during locomotion would require a device that can perturb the knee without impeding natural movements, which is at best difficult. Therefore, an approach that does not hinder natural movements is needed. A few such methods have been suggested for the upper limb, though these require subject-specific calibration, often together with experimental measurements of stiffness [16], [17].

In this paper, we propose a method to estimate active knee joint stiffness using a detailed musculoskeletal model
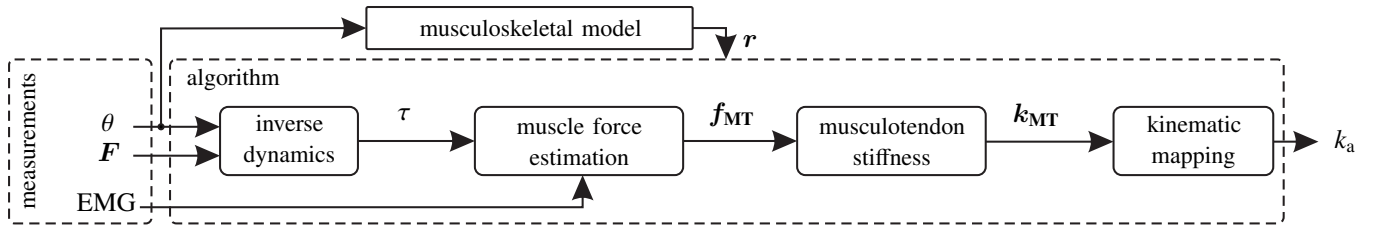
Fig. 1. Overview of method to determine active knee joint stiffness k_a. Two different methods are used for muscle force estimation: one is based on load sharing (Section II-C) and one is based on EMG (Section II-D). Inputs of the algorithm are knee flexion angle θ and force sensor or force-plate signals $\boldsymbol{F}$, or EMG signals. In the static case the inverse-dynamics block reduces to a simple kinematic transformation. Intermediate variables are: torque τ, musculotendon forces $\boldsymbol{f}_\text{MT}$, musculotendon stiffnesses $\boldsymbol{k}_\text{MT}$. The vector of moment arms $\boldsymbol{r}$ is used for muscle force estimation based on load sharing, and for the kinematic mapping from musculotendon stiffness to joint stiffness.

of the leg [18] and a parameterized model for short-range stiffness [19]. We also present initial results validating model performance during isometric conditions, and evaluating its sensitivity to the assumptions within the model. We also make initial estimates of how knee stiffness is modulated during gait, illustrating how this approach may eventually be used to obtain parameters relevant to the design of transfemoral prostheses for the restoration of locomotion.

II. Model

We propose a model-based method to estimate active knee joint stiffness without the need to apply perturbations to the joint (Fig. 1). A key requirement of the approach is the estimation of individual muscle forces. Many different combinations of muscle forces can produce the same torque, as the knee joint is spanned by many muscles. Two different methods are commonly used in the literature. Muscle forces are either estimated by distributing the joint torque among the muscles spanning a joint using optimization techniques [20], which is also called load sharing. Joint torque is usually found by inverse dynamics. The other commonly used method involves EMG recordings, which yield estimates of muscle activity from which muscle force can be computed. We evaluate both methods, as both have their advantages and disadvantages; load sharing typically fails to predict any co-contraction, and EMG-based estimates usually fail to estimate muscle forces that result in the measured joint torque.

A. Musculoskeletal Model

We used the lower-limb model developed by Arnold et al. [18], which features seven degrees of freedom and 43 muscles. We only considered the twelve muscles spanning the knee joint (listed in Section II-D). Parameters of the model like maximal isometric force, pennation angle, and optimal fiber length are based on a parameter study examining 35 cadavers [21]. It is implemented in OPENSIM, an open-source software for musculoskeletal modeling and simulations [22]. The input of the model is the knee flexion angle, the outputs are the moment arms of each muscle.

B. Musculotendon-Stiffness Model

The musculotendon stiffness was modeled as muscle stiffness in series with tendon stiffness, as commonly done in the literature (e.g. by Morgan [23]). Tendon stiffness was modeled based on a dimensionless force-strain curve as proposed by Zajac [24] and implemented by Delp et al. [25]. The parameters were taken from the lower-limb model [18]. As described in the introduction, we modeled the muscle's short-range stiffness, which has been shown to be proportional to the muscle force and inversely proportional to the optimal fiber length [19], [26]. We used the proportionality constant identified by Cui et al. based on measurements with feline muscles [19] which has already been successfully applied to the human arm [27].

C. Muscle Force Estimation Based on Load Sharing

Knee joint torque was determined using conventional inverse dynamics. To estimate individual musculotendon force we used a static optimization based on the min-max objective function, which in the case of a single degree of freedom joint reduces to an analytical solution: All muscles which cannot actively contribute to the observed torque (antagonists) will be set to zero; the active muscles (agonists), which are either the four extensors or the eight flexors in the model, are equally activated [28]. We compared the results to results obtained by minimizing the sum of squared muscle forces [29]. In both cases we used the normalization factor $N_i = f_{\text{M},i}^0 \cdot \cos \alpha_i$ for each musculotendon unit i, where $f_{\text{M},i}^0$ is its maximum isometric force, and α_i is its pennation angle.

We also estimated the muscle activity a_LS corresponding to the forces described above so that the results of the load sharing algorithm could be compared to the experimentally measured EMG, which is our proxy for muscle activity. This was done simply by dividing the estimated muscle force by the maximum force attainable by that muscle, as defined in our model [18]. This normalized measure of muscle activation was then compared to the EMG-based estimate, described below.

D. Muscle Force Estimation Based on EMG

As an alternative to load sharing (Section II-C) an EMG-based approach was used to estimate muscle forces. In contrast to load sharing, which relies on an inverse dynamic model to obtain joint torque, the EMG-based approach directly operates on the level of muscle activations. The downside of the latter is large measurement noise. Transcutaneous EMG of seven easily accessible muscles (rectus femoris (rf), vastus lateralis

(vl), vastus medialis (vm), semitendinosus (st), biceps femoris long head (blh), gastrocnemius medialis (gm), gastrocnemius lateralis (gl)) was recorded. The remaining five muscles in the model spanning the knee joint, which are less easy to access, were estimated similar to Barrett et al. [30]: the vastus intermedius (vi), the biceps femoris short head (bsh), the semimembranosus (sm), the gracilis (gr) and the sartorius (sr). The respective activations a were: $a_{vi} = 0.5 \cdot (a_{vm} + a_{vl})$, $a_{bsh} = a_{blh}$, $a_{sm} = a_{st}$, $a_{gr} = a_{sr} = 0.5 \cdot (a_{blh} + a_{st})$. EMGs were sampled at $1200\,Hz$ after an analog bandpass filter between $5\,Hz$ and $500\,Hz$. The recorded EMG was rectified and filtered using an RMS filter with window size $200\,ms$. A delay of $50\,ms$ accounted for the delay between EMG signal and muscle force. The signals were normalized to values obtained during maximum voluntary contractions (MVC), yielding the EMG-based estimate of muscle activity a_{EMG}. These estimates for each muscle i were multiplied by $f_{M,i}^0 \cdot \cos \alpha_i$ to obtain the muscle force; $f_{M,i}^0$ is the muscle's maximum isometric force and α_i is its pennation angle (values from the literature [21]).

III. Evaluation Methods

A. Load-Sharing Accuracy

As an indication of the accuracy with which muscle activity can be estimated, we quantified the discrepancy between the two fundamentally different procedures, load sharing (Section II-C) and EMG measurements (Section II-D).

Both computational procedures were evaluated on a single experimental data set. In these experiments six subjects performed isometric contractions while seated in a device that allows measurement of knee torque (described previously [31]); their foot was fixed and the knee angle was between $80°$ and $86°$. Subjects were required to maintain constant knee torques of either $15\,\%$ or $30\,\%$ of their maximum voluntary contraction in extension and flexion. Contractions at each level were maintained for six seconds; two measures were obtained at each level. The first and last second of each trial were discarded and only the four middle seconds were analyzed.

B. Model Validation under Isometric Conditions

To validate our model-based estimates of knee stiffness, we also obtained perturbation-based estimates under isometric measurements. Data were collected from four subjects. All procedures were approved by the Institutional Review Board at Northwestern University.

Perturbations were applied by a rotary torque motor configured as a rigid position servo. Subjects were attached to the motor using a custom-made thermoplastic cast extending from the toes to just below the knee. The knee was maintained at a flexion angle of $\theta = 60°$, and stochastic perturbations with a bandwidth of $7\,Hz$ and a standard deviation of $0.5°$ were applied for the purpose of stiffness estimation. Experimental trials lasted for 60 seconds, during which subjects were instructed to produce constant knee flexion torques ranging from $-40\,Nm$ up to $40\,Nm$ in steps of $10\,Nm$.

Mechanical impedance was estimated nonparametrically [3]. The responses, which were second-order, were then parameterized by the following equation.

$$\frac{\tau(s)}{\theta(s)} = Is^2 + bs + k$$

The parameter I is inertia, b viscosity, and the static gain k corresponds to elasticity or stiffness, which we compared to model estimates. The average stiffness when no torque was exerted by the subjects, the passive joint stiffness k_p, was subtracted from all measurements, because the model only predicts active joint stiffness k_a.

The experimentally identified values of the stiffness $k = k_a + k_p$ were compared to model-based estimates in matched conditions (fixed flexion angle $\theta = 60°$ at different flexion and extension torques). Only muscle forces obtained through load sharing were considered, since EMGs were not available for this data set. Specifically, the min-max criterion described in Section II-C was used. Muscle parameters were taken from our model described above [18], not specified for each subject in our experiments.

C. Stiffness during Gait

We also made initial attempts to estimate stiffness during gait. This was computed over five gait cycles for one healthy male subject (28 years old, $70\,kg$, $180\,cm$). The data used for these estimates was obtained during level-ground walking. It contained kinematic data (hip, knee and ankle angles) obtained with an optical tracking system (Vicon), kinetic data obtained using inverse dynamics and force platform measurements (Kistler), and EMG-measurements from six muscles; the same muscles as described in Section II-D, but without the rectus femoris. The activity of the rectus femoris was assumed to be equal to the mean of vastus medialis and vastus lateralis, analog to the method already applied for the hardly accessible muscles [30]. Resulting stiffness based on load sharing (Section II-C) and based on EMG (Section II-D) was compared.

IV. Results

A. Load-Sharing Accuracy

There were substantial differences between the muscle activations estimated from the load-sharing and EMG algorithms (Fig. 2). This was quantified by the standard deviation of these differences across all muscles and subjects. This measure was high and increased slightly with increasing activation. For the min-max criterion the standard deviation was $6.9\,\%MVC$ (mean value $1.4\,\%MVC$) for the tasks where subjects had to exert $15\,\%$ of their maximal torque capacity, and $9.4\,\%MVC$ (mean value $3.7\,\%MVC$) for the $30\,\%$-torque tasks. Results for load sharing based on the sum of normalized squared muscle forces were slightly worse (mean $3.6\,\%MVC$, std. dev. $8.5\,\%MVC$ for $15\,\%$ torque, mean $7.8\,\%MVC$, std. dev. $12.7\,\%MVC$ for $30\,\%$ torque). Based on these results we performed a sensitivity analysis to investigate influence of these different estimates of muscle force distribution on knee stiffness, as described in the following section.
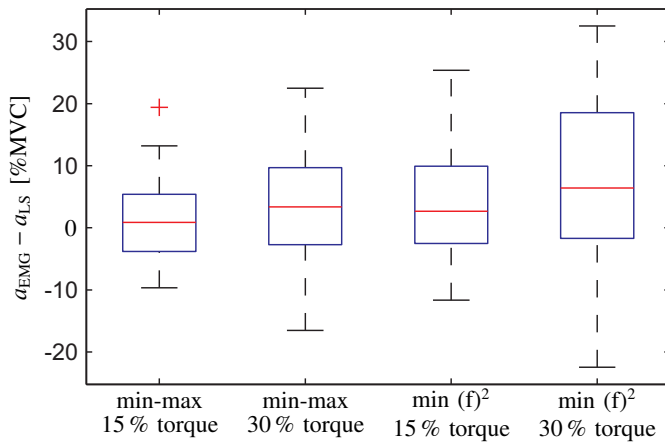
Fig. 2. Load-sharing accuracy for the min-max criterion and the cost function based on the sum of squared normalized muscle forces (min (f)^2). The difference between the activation obtained using load sharing (a_{LS}) and EMG-based activation (a_{EMG}) is shown separately for the tasks using 15 % torque level and 30 % torque level (Section III-A).
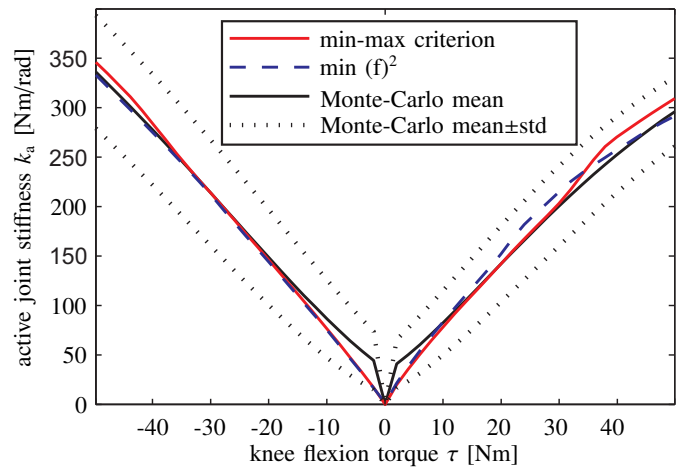

Fig. 3. Active knee joint stiffness at knee flexion angle of $\theta = 60°$ for different knee torque levels evaluated using different muscle activations. Stiffness resulting from load sharing based on the min-max criterion and load sharing based on the sum of squared normalized muscle forces (min (f)^2) is shown, together with mean and standard deviation of the Monte-Carlo analysis described in Section IV-B.

B. Sensitivity of Stiffness to Load-Sharing Accuracy

A Monte-Carlo analysis was used to investigate the sensitivity of joint stiffness estimates to the distribution of muscle forces obtained using the EMG-based approach and the min-max load-sharing algorithm. Model-based estimates of stiffness were determined for a torque range from -50 Nm to 50 Nm flexion torque in steps of 2 Nm; the knee angle was fixed at 60° for all simulations. Muscle activations were estimated using load sharing with the min-max criterion, which sets the antagonistic muscles to zero. Random errors were added to the activations of the agonist muscles to simulate the range of differences that could be obtained from kinetic and EMG-based estimates of muscle force. The standard deviation of the activation errors was obtained from the results presented in Fig. 2 as follows. Because we observed increasing errors with increasing activity, a linear model of the standard deviation of $a_{EMG} - a_{LS}$ was fit to the data. This model of the error standard deviation was used to generate simulated muscle activations with a normal distribution about the min-max estimate; the resulting muscle activations were not constrained to produce the original knee torque. We performed 1000 simulations using these random activation errors.

Overall, the joint stiffness estimates were relatively insensitive to changes in muscle activation. The greatest variation in predicted knee stiffness was at low force levels. This is likely due to the nature of our Monte Carlo simulations, which incorporated a model of activation uncertainty that was high even at low muscle forces. The standard deviation of the estimated knee stiffness at 10 Nm of knee flexion was 32.6 Nm/rad, which is 41.7 % of the corresponding min-max estimate (Fig. 3). In contrast, the standard deviation of the modeled activation error was 117.8 % of the min-max activation estimate at this same knee torque. This corresponds to a sensitivity of approximately 37 %. At 50 Nm of knee flexion, the standard deviation of the stiffness estimates was 11.3 %, and that of the muscle activation was 34.7 % of the

activation level. The sensitivity at these higher forces is similar (33 %), yet the accuracy of the stiffness estimation is greater due to the smaller range of relative muscle activations.

C. Model Validation under Isometric Conditions

The knee stiffness measured during isometric conditions increased similar to that predicted by the model, though the prediction accuracies differed between flexion and extension. Only active stiffnesses were compared, since our model does not predict passive joint stiffness. The experimentally measured passive stiffness ranged from $37 - 57$ Nm/rad across the four tested subjects. Model estimates reproduced experimentally identified stiffness better for extension torques than for flexion torques (Fig. 4); on average, excluding the zero torque conditions, experimentally measured stiffness was 17 % (std. dev. 9 %) lower than model estimates for extension torques, and 42 % (std. dev. 13 %) lower for flexion torques.

D. Stiffness during Gait

The magnitudes of the active knee stiffness during gait estimated based on load sharing and based on EMG were remarkably similar with a maximum of approximately 600 Nm/rad (Fig. 5). This result is consistent with our finding that stiffness magnitude is relatively insensitive to changes in muscle load sharing (Fig. 3). However, there was a substantial difference in timing; stiffness based on EMG increased prior heel strike, most likely due to preparatory co-contraction, while stiffness based on load sharing did not increase until after heel strike, due to its inability to estimate muscle co-contraction. This is probably also the reason why the minimal values of stiffness estimates based on load sharing were substantially lower than EMG-based estimates.
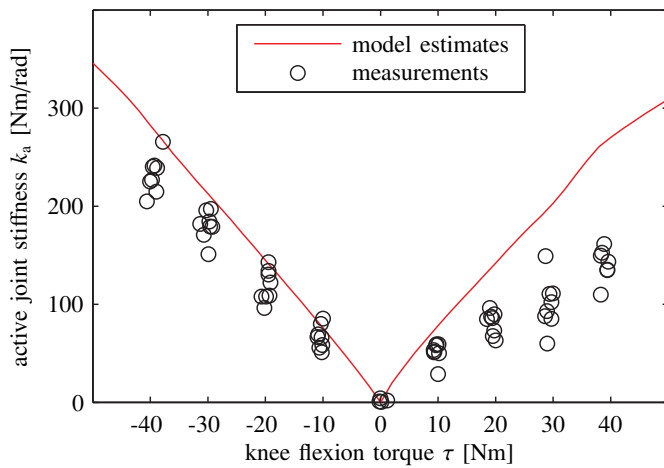
Fig. 4. Active knee joint stiffness at knee flexion angle of $\theta = 60°$ for different knee torque levels. The continuous line represents model estimates, circles represent stiffness identified from perturbation experiments from four subjects. Average passive stiffness k_p was experimentally determined at zero knee torque and subtracted from all measurements in order to compare to the model estimates of active knee joint stiffness ($k_a = k - k_p$).
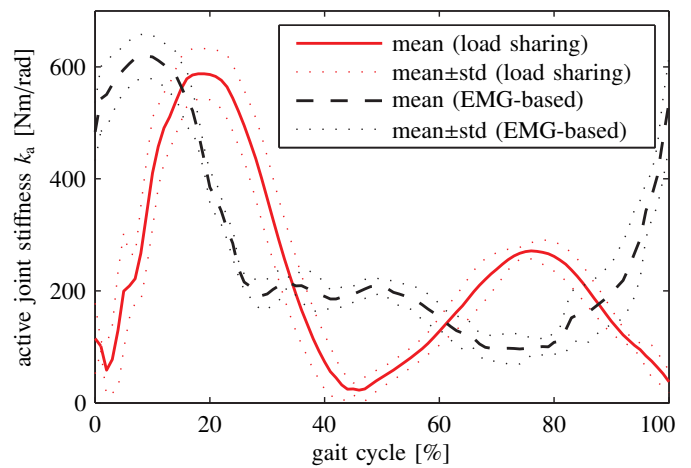

Fig. 5. Active knee joint stiffness for a healthy subject during level-ground walking evaluated using conventional load sharing (Section II-C) and EMG-based muscle force estimation (Section II-D). Mean and standard deviation of five gait cycles (from heel strike to heel strike) are depicted.

V. DISCUSSION

The objective of this study was to develop a model-based method for estimating knee stiffness during locomotion. We have developed such a model, which predicts active stiffness based on kinematic parameters of the knee and the force-dependent short-range stiffness of the muscles acting about the knee. Our initial results demonstrate that model predictions in isometric conditions are robust with respect to different approaches for estimating muscle force, and that these predictions are consistent with many features of stiffness measurements involving joint perturbations. These results also highlighted areas for improvement, which are described below.

In our isometric validation, estimated values of stiffness for knee extension torques were very close to experimentally determined values (Fig. 4). However, the model overestimated stiffness for knee flexion torques. This could be partly explained by errors in load sharing, though stiffness estimates during flexion fall outside of our confidence intervals related to load-sharing accuracy (Section IV-B). Previous experimental findings have suggested that active knee stiffness changes substantially with joint angle [7]. Our previous work has demonstrated that model-based estimates of arm stiffness are also sensitive to changes in joint angles and to muscle moment arms [27]. We have yet to complete similar sensitivity studies for the knee, though we would expect similar findings. This would suggest that precise angle measurements and subject-specific model parameters could lead to improved accuracy.

Our model-based and experimental results were similar to the two previous studies reported in the literature. Zhang et al. [7] demonstrated a similar torque-dependent increase in knee stiffness to that shown in this manuscript, though stiffnesses reported by Zhang were approximately 50 % smaller than our estimates. The discrepancy is likely due to the difference in perturbation amplitudes used in the two stud-

ies, since muscle short-range stiffness is known to decrease with increasing perturbation amplitude [8]; the amplitudes used by Zhang et al. were 140 % larger than ours. These amplitude-dependent effects on stiffness estimates have also been reported at the knee [15]. Larger amplitudes also may evoke substantial reflexes, which are not explicitly represented in our model. However, if such reflexes did contribute to knee stiffness during the continuous perturbations used to estimate stiffness, they would do so via a continuous change in muscle activation and force. Such steady-state changes would be captured in our current modeling approach. Nevertheless, the role of transient reflexes could be quite important during locomotion, and is something that will be considered in future validations of model performance during locomotion.

The stiffness estimates during gait are remarkably similar in magnitude for both the EMG-based approach and the load-sharing approach. The difference in timing, however, was substantial. It is much bigger than what could be expected based on our sensitivity analysis of stiffness to load-sharing errors in isometric conditions. This highlights the need to validate how well these approaches predict muscle force over time, which we have not attempted to incorporate in our current version of the model. EMG-based estimates may be improved by incorporating a more complex EMG-to-force processing [32]; a velocity-dependent component [33] might further improve estimates, as could a calibrated delay between EMG signal and force onset, which has been shown to be influenced by many parameters [34]. EMG-based estimates also can be improved by combining with load sharing estimates to ensure that the muscle activations and joint torques are matched throughout the gait cycle [35]. We are currently evaluating such approaches to obtain reliable estimates of stiffness during gait, with the aim to incorporate these in variable-impedance control of transfemoral prostheses. In this context, the role of joint viscosity will also have to be addressed, especially its relative importance compared to joint elasticity.

VI. Conclusion

We have developed a model-based approach that allows quantitative assessment of active knee joint stiffness without the need for applying perturbations to the joint. Our initial work has focused on quantifying the accuracy of the model-based estimates and identifying how they may be improved. Though there is still work to be done, the model can provide the first estimates of knee stiffness during locomotion. These estimates are essential for understanding how the mechanics of the leg contribute to physiological gait, and for understanding how to design robotic prostheses that can begin to replicate that gait in amputees.

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